

MAXIMUM LIKELIHOOD WITH HEURISTIC DETECTORS IN LARGE MIMO SYSTEMS FOR EFFECTIVELY RECEIVING BITS

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ABSTRACT

We propose low-complexity detectors for large MIMO systems with QAM constellations. These detectors work at the bit level and consist of three stages. In the first stage, ML decisions on certain bits are made in an efficient way. In the second stage, soft values for the remaining bits are calculated with multi iteration concept. In the third stage, these remaining bits are detected by means of a heuristic programming method for high-dimensional optimization that uses the soft values ("soft-heuristic" algorithm). We propose two soft-heuristic algorithms with different performance and complexity. We also consider a feedback of the results of the third stage for computing improved soft values in the second stage.

KEYWORDS: Genetic Algorithm, Heuristic Programming, ICI Mitigation, Large MIMO Systems, MIMO Detection, Multiple-Input Multiple-Output Systems, OFDM

1. INTRODUCTION (MIMO)

MULTIPLE-INPUT/MULTIPLE-OUTPUT (MIMO) systems for wireless communications have received considerable interest. A MIMO system with input dimension N_t and output dimension N_r can be described by the input-output relation

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \quad (1)$$

Where $\mathbf{s} \in \mathcal{S}^{N_t}$ is the transmit symbol vector (here, \mathcal{S} denotes a finite symbol alphabet $\mathbf{y} \in \mathcal{C}^{N_r}$ is the received vector $\mathbf{H} \in \mathcal{C}^{N_r \times N_t}$, is the channel matrix, and $\mathbf{n} \in \mathcal{C}^{N_r}$ is a noise vector. The MIMO model is relevant to multi antenna wireless systems, orthogonal frequency-division multiplexing (OFDM) systems and code-division multiple access (CDMA) systems. Here, we consider the detection of \mathbf{s} from \mathbf{y} under the frequently used assumptions that the channel matrix \mathbf{H} is known and the noise is \mathbf{n} independent and identically distributed (iid) circularly symmetric complex Gaussian, $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_{N_r})$ where σ_n^2 is the noise Variance \mathbf{I}_{N_r} and is the $N_r \times N_r$ identity matrix.

State of the Art

The result of maximum-likelihood (ML) detection, which minimizes the error probability for equally likely transmit

Vectors, $\mathbf{s} \in \mathcal{S}^{N_t}$ is given by [1]

$$\hat{\mathbf{s}}_{ML}(\mathbf{y}) = \underset{\mathbf{s} \in \mathcal{S}^{N_t}}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{H}\mathbf{s}\| \quad (2)$$

ML detection is infeasible for larger MIMO systems because its computational complexity grows exponentially with N_t . This is also true for efficient implementations of ML detection using the sphere-decoding algorithm. Among the suboptimum detection methods, linear equalization methods often perform poorly because each symbol is quantized individually. Detection using decision-feedback equalization, also known as nulling-and-canceling (NC), outperforms linear equalization but is still inferior to ML detection. NC implementations with reliability-based symbol ordering include V-BLAST and dynamic NC. Detectors based on lattice reduction have polynomial average complexity and tend to outperform equalization-based detection. Detectors on semi definite relaxation (SDR) exhibit excellent performance but are significantly more complex than Equalization-based detectors. The “subspace marginalization with interference suppression” (SUMIS) soft-output detector has a low and fixed (deterministic) complexity.

The suboptimum multistage detectors proposed can achieve near-optimum performance with a complexity much lower than that of sphere decoding. A survey of MIMO detection using heuristic optimization (or programming) methods, such as genetic algorithms, short-term or reactive search, simulated annealing, particle swarm optimization. In particular, several adaptations of genetic algorithms to MIMO detection have been proposed recently, large MIMO systems with several tens of antennas have attracted increased attention due to their high capacity. Suboptimum detection methods for large MIMO systems include local search algorithms such as likelihood ascent search (LAS) and reactive search, as well as a belief propagation algorithm.

Contribution

Extending our work, we present low-complexity detectors for large MIMO systems using a BPSK or QAM symbol alphabet.

The proposed MIMO detectors operate at the bit level and consist of three stages as depicted in Figure 1. The first stage performs *partial ML detection*. Let the bit vector $\hat{\mathbf{b}}_{\text{ML}} = (\hat{\mathbf{b}}_{\text{ML},k})$ denote the ML solution at bit level that corresponds to $\hat{\mathbf{S}}_{\text{ML}}$ as described. In the first stage, certain bits are calculated by means of the iterative algorithm presented.

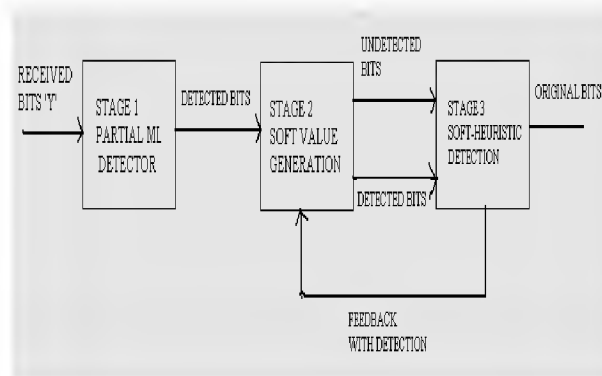


Figure 1: Block Diagram of the Proposed MIMO Detector and Denote the Sets of Indices of Respectively, the Detected and Undetected Bits at a Given Iteration

We reformulate that algorithm in terms of lower and upper bounds that also play an important role in the following stages. (We note that in contrast, where a single-input single-output system with inter symbol interference was considered and the undetected bits were subsequently detected using a linear or decision-feedback equalizer, here we consider a MIMO system and replace the equalizer by a novel bit-level detector consisting of the second and third stages shown in Figure 1). In the second stage, *soft values* β_k for the undetected bits are calculated from the lower and upper

bounds. In the third stage, the undetected bits are detected by means of an iterative “soft- heuristic” optimization algorithm that uses the ML bits $\mathbf{b}_{ML,k}$ and soft values β_k produced by the first two stages. We propose two soft-heuristic algorithms with different performance and complexity. Both algorithms are based on principles used to solve large-scale optimization problems and are therefore especially suitable for large MIMO systems. The *sequential soft-heuristic algorithm* is a soft-input version of the greedy optimization algorithm presented, however using an improved (no greedy) order of decisions inspired by the Nelder-Mead algorithm.

The *genetic soft-heuristic algorithm* is a soft-input and otherwise modified version of the genetic algorithm presented. It is substantially different from genetic algorithms previously proposed for MIMO detection in that it uses the results of the first two stages for an improved initialization and includes a local search procedure that produces improved candidate solutions even for very small population sizes. The reduced population sizes result in a low complexity and make the algorithm suited to large MIMO systems. In the sequential soft-heuristic algorithm, the bits detected by the third stage are fed back to the second stage in order to obtain improved soft values. A similar feedback can also be used with the genetic soft-heuristic algorithm.

II. PARTIAL ML DETECTION AND GENERATION OF SOFT VALUES

For a QAM symbol alphabet S , where $|S| = 2^B$ with an even

$B = \log_2 |S|$, there is a unique vector $\mathbf{V} = (v_1, \dots, v_B)^T \in \mathcal{C}^B$ such that every symbol $s \in S$ can be written

$$\mathbf{s} = \sum_m \mathbf{b}_m(s) = \mathbf{V}^T \mathbf{B}(s) \quad (3)$$

With a bit vector $\mathbf{b}(s) = (b_1(s), \dots, b_B(s))^T \in \{-1, 1\}^B$ that provides a unique binary representation of the symbol s . The complex vector \mathbf{v} only depends on $|S|$ e.g., $\mathbf{V} = (1 \ j)^T$ $|S| = 4$, $\mathbf{V} = (2 \ 1 \ 2j \ j)^T$, for $|S| = 16$, and $\mathbf{V} = (4 \ 2 \ 1 \ 4j \ 2j \ j)^T$ for $|S| = 64$. For $|S| \geq 16$, the binary representation defined is not a Gray mapping. Although BPSK is not a special case of QAM, it is nevertheless a (trivial) special case of $B=1$, $\mathbf{V}=(1)$, and $b_1(s)=s$. Let $s_p = (S)_p$ denote the p th element of. For QAM or BPSK, using for each s_p , a binary representation of the MIMO system in $\mathbf{y} = \mathbf{A}\mathbf{b} + \mathbf{n}$ Here, $\mathbf{A} = \mathbf{H} \times \mathbf{V}^T \in \mathcal{C}^{N_r \times B N_t}$ is an equivalent channel matrix $\mathbf{b} = \mathbf{b}(s) = (b^T(s_1) \dots b^T(s_{N_t}))^T \in \{-1, 1\}^{B N_t}$ is the binary representation of the transmit symbol vector. For BPSK, $\mathbf{b} = \mathbf{s}$ and $\mathbf{A} = \mathbf{H}$. The ML detection rule can now be equivalently formulated at the bit level as

$$\hat{\mathbf{b}}_{ML} = \arg \min_{\mathbf{b} \in \{-1, 1\}^{B N_t}} \|\mathbf{y} - \mathbf{A}\mathbf{b}\|^2 \quad (4)$$

Partial ML Detection

The first stage of the proposed MIMO detector computes *some* elements $\mathbf{b}_{ML,k}$ of the ML detection result $\hat{\mathbf{b}}_{ML}(\mathbf{y})$ in an efficient manner. This is done by means of the algorithm proposed, which will now be reviewed. In what follows, let $\mathbf{z} = \mathbf{A}^H \mathbf{y}$ and $\mathbf{G} = \mathbf{A}^H \mathbf{A}$. Furthermore, let $\mathcal{I} = \{1, \dots, B N_t\}$ denote the index set of the elements of $\mathbf{b} = (\mathbf{b}^T(s_1) \dots \mathbf{b}^T(s_{N_t}))^T = (b_1 \dots b_{B N_t})^T$, and denote by b_k, z_k , and $G_{k,l}$, with $k, l \in \mathcal{I}$, the elements of \mathbf{b} , \mathbf{z} , and \mathbf{G} , respectively. As explained in the following, we expand ML metric $\|\mathbf{y} - \mathbf{A}\mathbf{b}\|^2$ Metric with respect to a specific bit.

Generation of Soft Values

For detection of the bits, $k \in \mathcal{I}_{ML}$ that was not detected by the partial ML detection stage (Stage 1), we first generate *Soft values* (Stage 2). The soft values will constitute an Input to Stage 3. For a given, $k \in \mathcal{I}_{ML}$. We recall that for All such that. Because x_k is unknown except for the fact that $L_k(\mathbf{D}_{ML}) \leq x_k \leq U_k(\mathbf{D}_{ML})$, we model x_k as a random variable

that is uniformly distributed on $[L_k(D_{ML}), U_k(D_{ML})]$. We now define the soft value as the expected “soft decision”, can be viewed as the counter part of the hard decision that was made for $k \in D_{ML}$ in Stage 1. The soft values β_k can be easily calculated from the bounds $L_k(D_{ML})$ and $U_k(D_{ML})$ using the uniform distribution of x_k , we obtain Note that. The bounds and, thus, the soft values in are also valid if no ML bits are detected in Stage 1, but the tightness of the bounds and the quality of the soft values improve if more bits are detected.

III. THE SEQUENTIAL SOFT-HEURISTIC ALGORITHM

The task of Stage 3 is to determine the bits b_k , $k \in Q_{ML}$, bits are not detected in Stage 1. A linear or decision-feedback equalizer is used for this task. Here, for improved performance in large MIMO systems, we propose two alternative soft-input heuristic algorithms that make use of the soft values β_k , $k \in Q_{ML}$, computed in Stage 2. The *sequential soft-heuristic algorithm* (SSA) described in this section is a soft-input version of the greedy algorithm presented, a solution vector is generated in a bit-sequential (recursive) manner by detecting one β_k , $k \in Q_{ML}$, in each recursion step; the corresponding decision is never reconsidered. However, the SSA employs a different initialization that takes into account the results of Stages 1 and 2. Furthermore, it uses an improved (non greedy) order of decisions inspired by the Nelder-Mead algorithm. Finally, it performs a continuous update of the soft values via a feedback from Stage 3 to Stage 2.

Initialization

The greedy algorithm (adapted to our bit alphabet $\{1, -1\}$). In contrast, the initial input vector used by the SSA is composed of the ML bits $b_{ML,k}$, $k \in D_{ML}$, detected in Stage 1 and the soft values β_k , $k \in Q_{ML}$, calculated in Stage 2,

Statement of the SSA

Let $D \subseteq D_{ML}$ denote the index set of all bits b_k detected so far, which consist of the ML bits $b_{ML,k}$ (index set D_{ML}) and the suboptimum detection results obtained so far in the present Stage 3 (index set $D \setminus D_{ML}$). In each recursion step, the SSA detects one of the as yet undetected bits, The iterated vector that provides the input to the recursion step considered where the b_k , $k \in D$ are the bits detected so far and are soft values. The SSA now produces a modified vector in which the soft value contained in at some is replaced by a hard bit

Finally, the index sets are updated according to bits. It remains to determine the “best” index and the “best” bit value. Motivated, the greedy strategy chooses the and yielding the largest increase. The SSA takes a different approach that is inspired by the Nelder-Mead optimization algorithm. First producing the maximum *decrease* of are determined; then, this “worst decision” is inverted by setting the bit to the respective other value. As will be shown in Section V-E, this strategy yields a significantly better performance than the greedy strategy. An intuitive explanation might be that avoiding these worst decisions reduces error propagation. For a formal statement, we define the *gain function* which characterizes the increase in obtained by replacing the fact that yields

IV. THE GENETIC SOFT-HEURISTIC ALGORITHM

The *genetic soft-heuristic algorithm* (GSA) is an alternative to the SSA with better performance but higher complexity. It is a soft-input version of the genetic optimization algorithm presented, and differs from that algorithm in its initialization (which uses the results of Stages 1 and 2), the local search algorithm, and the mutation operation. Also, it contains a novel diversification operation that uses soft values, it adds to the genetic operations (crossover, mutation, selection and diversification a local search. A block diagram of the GSA with initialization is shown

Figure 2 the initialization procedure generates an initial start set of candidate solutions (CSs) for the first iteration of the GSA, using the ML bits, from Stage 1 and the soft values, from Stage 2. This is done in two steps: first, a *preliminary* initial start set is generated; next, this preliminary set is improved by a local search algorithm. In iteration of the GSA, the *crossover*, *mutation*, and *local search* steps use the locally optimized CSs, to calculate new CSs. Here, is assumed even for simplicity, with. In the *selection* step, identical CSs in the extended set consisting of the previous CSs and the additional CSs are removed, and the best CSs, those with the largest values are used as the start set for the next iteration. Hence, the number of CSs in each start set and, therefore, the complexity of each iteration are limited, whereas the quality of the CSs improves with progressing iterations. After a predetermined maximum number of iterations, the best CS in the current CS set is used as the final result of the GSA. Here, represents a tradeoff between performance and computing time. However, beyond a certain point, the performance cannot be improved further by increasing.

Generation of the Preliminary Initial Start Set

Each CS in the preliminary initial start set contains the ML bits and detected in stage 1.

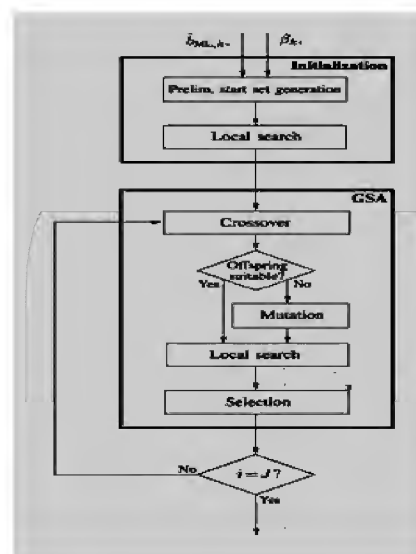


Figure 2: Block Diagram of the GSA with Initialization

The remaining bits are derived from the soft value calculation in stage 2 by means of following modified version $b_{1,k} = (b_{ML,k} \text{ when } k \in D_{ML}) \text{ or } (\text{sgn}(\beta_k), k \in Q_{ML})$ The First CS b_1 of the preliminary initial start set is generated by quantizing the soft values The remaining CSs are generated by interpreting the absolute values of the soft values as reliability measures and flipping unreliable bits. More precisely, let us denote the indices k with ordering according to increasing reliability, Then is formed by flipping the two most unreliable bits Similarly, is formed by flipping the four most unreliable bits. Continuing this way, for each new two more bits the most unreliable bits of those not flipped so far are flipped. The Elements of the last CS of the preliminary initial start set are $b_{\max,k} = (-b_{1,k} \text{ when } k \in \{k_1, k_2, \dots, k_{\max}\})$ thus given Here, b_1, b_2 are design parameter that satisfies and is determined empirically

Diversification

A performance improvement can be achieved by an optional diversification stage. As shown in Figure 3, this adds an outer loop to the GSA. Let superscript denote the iteration index for this outer loop. Furthermore, let denote the CS set obtained at the outer iteration after termination of the (inner) GSA loop, at the output of the GSA's selection stage.

The diversification stage calculates from new soft values these soft values are then used by the initialization stage of the GSA to calculate a new preliminary initial start set for the next outer iteration.

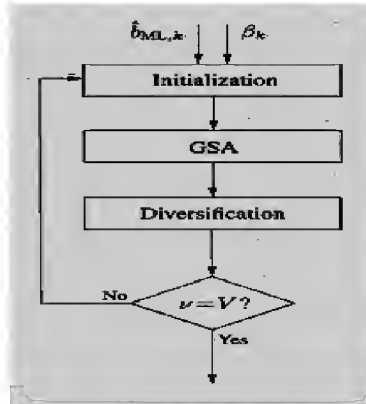


Figure 3: Block Diagram of the Extended GSA Including a Diversification Stage and an Outer Loop

The initialization stage is modified in that the first CS of this new preliminary initial start set is chosen as the best CS obtained from the previous outer iteration, i.e., the CS from with largest value. Assuming for concreteness that this best CS is remaining CSs are constructed by means of the scheme described in Section, using the new soft values. Let us again denote the indices k by ordered such that This outer loop iteration process is initialized at with the preliminary initial start set, based on the original soft values, The process is terminated after a predetermined maximum number of iterations. The best CS at that point, is used as the final result of the extended GSA. Alternatively, soft information for a soft-input channel decoder is computed. The performance improvements achieved by diversification will be demonstrated in Section

V. SIMULATION RESULTS

We present simulation results demonstrating the uncoded BER and computational complexity of the proposed detectors.

Simulation Scenarios and Parameters

Two scenarios are considered: a spatial-multiplexing multi antenna system and an OFDM system with inter carrier interference (ICI). In the spatial-multiplexing scenario, the channel matrix has iid Gaussian entries. In the OFDM/ICI scenario, the MIMO system corresponds to the transmission of a single OFDM symbol consisting of subcarriers over a doubly selective single-antenna channel, with ICI due to the channel's time variation. Thus, the dimension of the MIMO system main task of the MIMO detector is a mitigation of the detrimental effects of ICI. The doubly selective fading Channel is characterized by a Gaussian wide-sense stationary uncorrelated scattering (WSSUS) model with uniform delay and Doppler profiles (brick-shaped scattering function).

The maximum delay (channel length) is the cyclic prefix Length and the maximum Doppler frequency is 16% of the subcarrier spacing. Because, inter symbol interference is avoided. For each transmit symbol vector, a new channel realization was randomly generated using the method presented. The MIMO channel matrix depends on the impulse response of the doubly selective fading channel as well as the (rectangular) transmit and receive pulses as described. The entries of are not independent nor identically distributed; they exhibit a strong diagonal dominance and an approximate band structure, which leads to an approximate band structure. We compare the proposed detectors—hereafter

briefly termed “SSA” and “GSA”—with ML detection using the Schnorr-Euchner sphere decoder; the MMSE detector, the NC detector with MMSE nulling vectors and V-BLAST ordering using the efficient implementation described; an SDR-based detector with rank-one approximation; Three-stage LAS detector; and the SUMIS detector. We did not simulate existing genetic algorithms for MIMO detection, such as since they assume large populations and are therefore infeasible for large MIMO systems. MMSE detection, NC, and SUMIS require an estimate of the noise variance; however, the true value of was used in our simulations Finally, for GSA1, values varied from 0 dB to 20 dB in steps of 5 dB) at the beginning of iteration for a 64x64 spatial-multiplexing system using 4QAM.

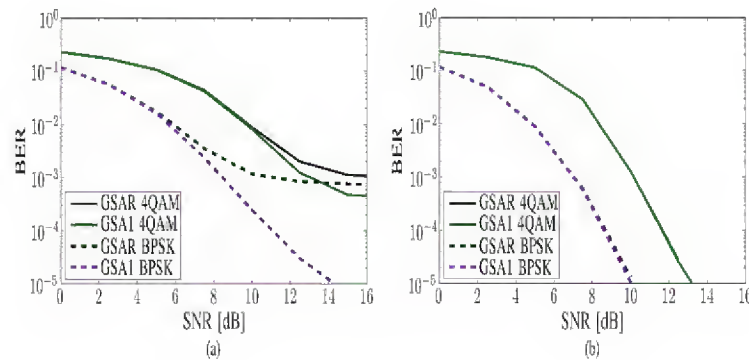


Figure 4: BER of GSA1 and GSAR for Spatial-Multiplexing Systems Using BPSK and 4QAM, of Dimension (a) 8 x8 and (b) 64x 64

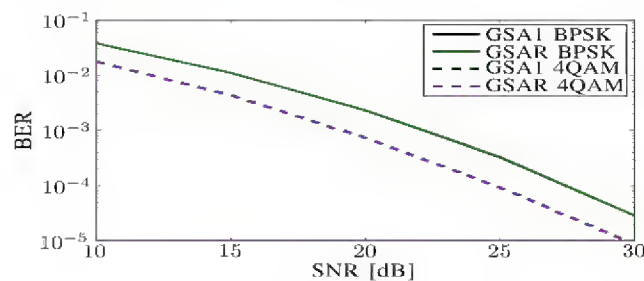


Figure 5: BER of GSA1 and GSAR for 64 64 OFDM/ICI Systems Using BPSK and 4QAM the GSA1 and GSAR Curves Coincide for Each of the Two Modulation Formats

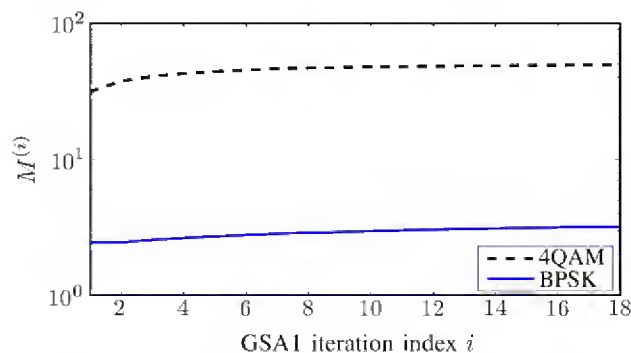


Figure 6: Average Number of Candidate Solutions at the Beginning of GSA1 Iteration for 64 64 Spatial-Multiplexing Systems Using BPSK and 4QAM

For the BPSK system, the number of candidate solutions is rather small. This indicates that there are only few local maxima, and thus the “searching in parallel” approach of GSA1 cannot exploit its full potential. This agree, which shows that the performance advantage of GSA1 over SSA is very small. However, for the 16QAM system, the number of candidate solution is increased.

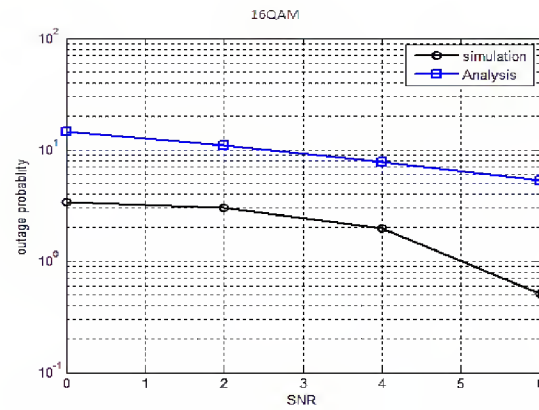


Figure 7: Simulation Analysis of SNR vs Outage Probability

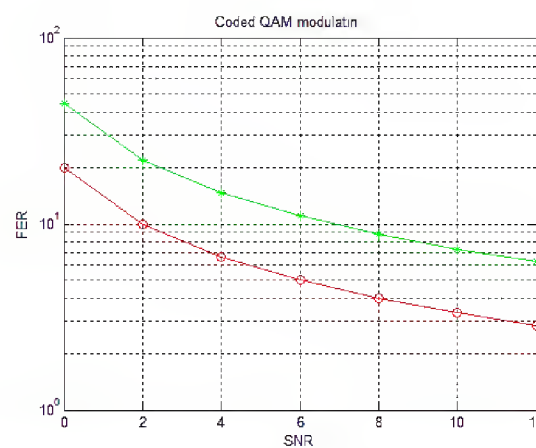


Figure 8: Simulation Result for SNR vs FER

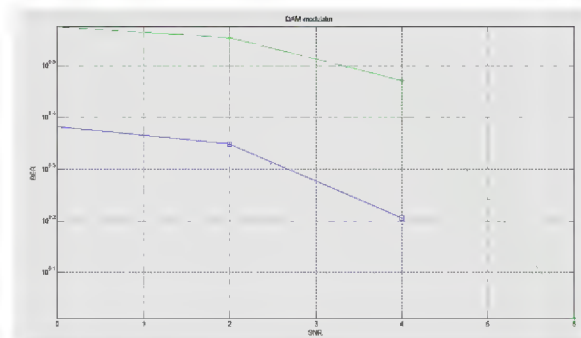


Figure 9: Simulation Result for SNR vs BER

Table 1: Computational Complexity of Various DETECTORS for OFDM/ICE SYSTEMS Using 16QAM

VECTOR COMPLEXITY (kilops per received vector)									
$N_t = N_r$	MMSE	NC	LAS	SUMS	SSA	GSA1	GSA3	GSAR	ML
8	1	1	28	7	2	2	4	10	13
16	3	5	222	41	9	9	16	70	3061
32	12	20	1800	295	34	42	944	564	-
64	49	81	14600	2228	135	302	8458	4947	-

Input dimensional $N_r = N_t$ is 128bits, from this soft heuristics algorithm we can get efficient output with reduced bit error rate and get efficient bits.

VI. CONCLUSIONS

We presented low-complexity bit-level detectors for MIMO systems employing a QAM constellation. The detectors combine efficient partial ML detection, generation of soft values, and a novel type of suboptimum detection based on heuristic optimization and soft values ("soft-heuristic optimization"). We proposed two alternative soft-heuristic algorithms, the sequential soft-heuristic algorithm (SSA) and the genetic soft-heuristic algorithm (GSA). Due to their architecture and their use of efficient techniques for high-dimensional optimization, the SSA and GSA are especially advantageous for large MIMO systems. Moreover, their ability to exploit diagonal dominance of the channel matrix for a complexity reduction makes them attractive for ICI mitigation in OFDM systems.

We evaluated the performance of the SSA and GSA for spatial-multiplexing multi antenna systems and OFDM/ICI systems. In spatial-multiplexing systems using BPSK, the SSA and GSA outperform MMSE detection and nulling-and-canceling (NC) and perform similar as semi definite relaxation (SDR) based detection, likelihood ascent search (LAS) based detection, and the SUMIS detector. For 16QAM, the SSA fails to perform satisfactorily whereas the GSA outperforms MMSE, NC, SDR, LAS, and SUMIS at low-to-medium SNRs and, for larger systems, at all considered SNRs. The SSA is less complex than SDR and LAS and has a better scaling behavior, whereas the GSA has a higher complexity than the other suboptimum detectors. In OFDM/ICI systems, the SSA and GSA significantly outperform MMSE detection. Similarly to NC, SDR, LAS, and SUMIS, they achieve effectively optimum (ML) performance for BPSK and 16QAM. Furthermore, they are significantly less complex than all other suboptimum detectors considered except MMSE and NC.

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